Lecture 3: Dynamics of small open economies

Open economy macroeconomics, Fall 2006

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Dynamics of small open economies

- Required readings: OR chapter 2.1–2.3

- Supplementary reading: Obstfeld, M and K. Rogoff (1995): "The intertemporal approach to the current account" in Grossman and Rogoff (eds): Handbook of International Economics, vol 3 (or NBER Working Paper No. 4893, 1994).

An infinite-horizon small open economy model

- Production function

$$Y_s = A_s F(K_s)$$

- Capital accumulation (no depreciation)

$$K_{s+1} = I_s + K_s$$

- Intertemporally additive preferences

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} u(C_s) = u(C_t) + \beta u(C_{t+1}) + \beta^2 u(C_{t+2}) + \cdots$$

- Deriving the intertemporal budget constraint:

  – Period by period budget constraint

$$
\begin{aligned}
CA_t &= B_{t+1} - B_t = Y_t + rB_t - C_t - I_t - G_t \\
& \text{or} \\
(1+r)B_t &= C_t + I_t + G_t - Y_t + B_{t+1} \qquad (*)
\end{aligned}
$$

  – Forward by one period and divide by $1+r$

$$
B_{t+1} = \frac{C_{t+1} + I_{t+1} + G_{t+1} - Y_{t+1}}{1+r} + \frac{B_{t+2}}{1+r}
$$

  and substitute back in equation (*)

$$
\begin{aligned}
(1+r)B_t &= C_t + I_t + G_t - Y_t \\
&\quad + \frac{C_{t+1} + I_{t+1} + G_{t+1} - Y_{t+1}}{1+r} + \frac{B_{t+2}}{1+r}
\end{aligned}
$$

  – Repeat process to eliminate B_{t+2}

$$
B_{t+2} = \frac{C_{t+2} + I_{t+2} + G_{t+2} - Y_{t+2}}{1+r} + \frac{B_{t+3}}{1+r}
$$

$$
\begin{aligned}
(1+r)B_t \quad = \quad & C_t + I_t + G_t - Y_t \\
& + \frac{C_{t+1} + I_{t+1} + G_{t+1} - Y_{t+1}}{1+r} \\
& + \frac{C_{t+2} + I_{t+2} + G_{t+2} - Y_{t+2}}{(1+r)^2} \\
& + \frac{B_{t+3}}{(1+r)^2}
\end{aligned}
$$

– Intertemporal budget constraint

$$
\sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} (C_s + I_s) = (1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} (Y_s - G_s)
$$

where we have imposed the transversality condition

$$
\lim_{T \to \infty} \left(\frac{1}{1+r} \right)^{T} B_{t+T+1} = 0
$$

- Optimisation problem

$$\max \sum_{s=t}^{\infty} \beta^{s-t} u((1+r)B_s - Y_s - I_s - G_s - B_{s+1})$$

  - First-order condition with respect to B_{s+1}

$$\begin{aligned} -\beta^{s-t}u'(C_s) + \beta^{s+1-t}u'(C_{s+1})(1+r) &= 0 \\ u'(C_s) &= \beta(1+r)u'(C_{s+1}) \end{aligned}$$

  - Note that only if $\beta = 1/(1+r)$ is a constant steady-state consumption path optimal

    * If $\beta < 1/(1+r)$ consumption shrinks forever

    * If $\beta > 1/(1+r)$ consumption grows forever

– How to get constant steady-state consumption with $\beta \neq 1/(1+r)$?

  * Make discount factor β a function of consumption ($\beta'(C_t) < 0$)

  * Overlapping-generations (OLG) model

- Consumption functions: some special cases

1. $\beta = 1/(1+r) \Longrightarrow C_t = C_{t+1} = C_{t+2} = \cdots = C_s$

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} C_t = (1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (Y_s - G_s - I_s)$$

$$\frac{1}{1-\frac{1}{1+r}} C_t = (1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (Y_s - G_s - I_s)$$

$$C_t = \frac{r}{1+r} \left[(1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (Y_s - G_s - I_s)\right]$$

$$C_t = \frac{r}{1+r} W_t$$

i.e.; optimal consumption C_t equals the annuity value of total discounted wealth net of government spending and investment

2. Isoelastic utility $u(C) = \frac{C^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$

– Euler equation

$$u'(C) = C^{-\frac{1}{\sigma}}$$

$$\begin{aligned} C_t^{-\frac{1}{\sigma}} &= \beta(1+r)C_{t+1}^{-\frac{1}{\sigma}} \\ C_{t+1} &= \beta^{\sigma}(1+r)^{\sigma}C_t \\ C_{t+2} &= \beta^{\sigma}(1+r)^{\sigma}C_{t+1} \\ &= \left(\beta^{\sigma}(1+r)^{\sigma}\right)^2 C_t \\ & \dots \\ C_s &= \left(\beta^{\sigma}(1+r)^{\sigma}\right)^{s-t} C_t \end{aligned}$$

– Insert into intertemporal budget constraint

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}C_s = (1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - G_s - I_s)$$

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(\beta^{\sigma}(1+r)^{\sigma})^{s-t}C_t = (1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - G_s - I_s)$$

$$C_t\sum_{s=t}^{\infty}\left(\beta^{\sigma}(1+r)^{\sigma-1}\right)^{s-t} = (1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - G_s - I_s)$$

$$C_t = \frac{(1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - G_s - I_s)}{\sum_{s=t}^{\infty}\left(\beta^{\sigma}(1+r)^{\sigma-1}\right)^{s-t}}$$

– Noting that if $\beta^\sigma(1+r)^{\sigma-1} < 1$

$$\begin{aligned}\sum_{s=t}^{\infty}\left(\beta^\sigma(1+r)^{\sigma-1}\right)^{s-t} &= \frac{1}{1-\beta^\sigma(1+r)^{\sigma-1}} \\ &= \frac{1+r}{r+\underbrace{1-\beta^\sigma(1+r)^\sigma}_{\vartheta}}\end{aligned}$$

then

$$C_t = \frac{r+\vartheta}{1+r}\left[(1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - G_s - I_s)\right]$$

- A fundamental current account equation

  - The permanent level of a variable X_t is the hypothetical constant level of the variable that has the same present value as the variable itself

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\widetilde{X}_t = \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}X_s$$

$$\widetilde{X}_t \equiv \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}X_s$$

– Assume $\beta = \frac{1}{1+r}$

$$
\begin{aligned}
CA_t &= B_{t+1} - B_t \\
&= Y_t + rB_t - C_t - G_t - I_t \\
&= Y_t + rB_t - G_t - I_t \\
&\quad \underbrace{-\frac{r}{1+r}\left[(1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - G_s - I_s)\right]}_{C_t} \\
&= Y_t + rB_t - rB_t - G_t - I_t - \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} Y_s \\
&\quad + \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} G_s + \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} I_s \\
&= \left(Y_t - \widetilde{Y}_t\right) - \left(G_t - \widetilde{G}_t\right) - \left(I_t - \widetilde{I}_t\right)
\end{aligned}
$$

– Economy runs a current account deficit (surplus) if output is below (above) its permanent level, government spending is above (below) its permanent level or investment is above (below) its permanent level

– What if $\beta \neq \frac{1}{1+r}$?

– Assume isoelastic preferences

$$\begin{aligned} CA_t &= Y_t + rB_t - C_t - G_t - I_t \\ &= Y_t + rB_t - G_t - I_t \\ &\quad \underbrace{-\frac{r+\vartheta}{1+r}\left[(1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - G_s - I_s)\right]}_{C_t} \\ &= Y_t + rB_t - G_t - I_t \\ &\quad -\frac{r}{1+r}\left[(1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - G_s - I_s)\right] \\ &\quad -\frac{\vartheta}{1+r}\underbrace{\left[(1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - G_s - I_s)\right]}_{W_t} \\ &= \left(Y_t - \widetilde{Y}_t\right) - \left(G_t - \widetilde{G}_t\right) - \left(I_t - \widetilde{I}_t\right) - \frac{\vartheta}{1+r}W_t \end{aligned}$$

where $\vartheta = 1 - \beta^{\sigma}(1+r)^{\sigma}$

- Current account driven by two distinct motives: pure *smoothing* motive (as in the case $\beta = \frac{1}{1+r}$) and a *tilting* motive
  - If $\vartheta < 0$ (country is relatively patient, $\beta > 1/(1+r)$), current account balance is increased
  - If $\vartheta > 0$ (country is relatively impatient, $\beta < 1/(1+r)$), current account balance is reduced

- An aside: debt sustainability or when is a country bankrupt?

  – Rearrange the intertemporal budget constraint

$$
\begin{aligned}
-(1+r)B_t &= \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - C_s - G_s - I_s) \\
&= \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} TB_s
\end{aligned}
$$

  where TB_s is the trade balance

  – Suppose that the economy grows at rate g

$$\frac{Y_{s+1}}{Y_s} = 1+g, \qquad g > 0$$

  and that

$$\frac{B_{s+1}}{B_s} = 1+g$$

  so that the debt-to-output ratio (B_s/Y_s) is constant

– The current account balance is

$$CA_s = B_{s+1} - B_s = gB_s = rB_s + TB_s$$

which implies

$$\frac{TB_s}{Y_s} = \frac{-(r-g)B_s}{Y_s}$$

that is; to maintain a constant debt-to-GDP ratio the country needs to pay only the difference between the real interest rate and the growth rate of the economy

A stochastic current account model

- Future levels of output, investment and government spending are stochastic

- Only asset is riskless bond which pays a constant real interest rate r

- Replace assumption of perfect foresight with assumption of rational expectations (Muth, 1961): agents' expectations are equal to the mathematical conditional expectation based on the economic model and all available information about current economic variables

  – Let $\mathcal{F}_t$ denote the information available to the agent at time t and let X_{t+1}^e be the agent's subjective expectation of a variable X_{t+1} made at time t

  – Muth's rational expectations hypothesis (REH) then implies

$$X_{t+1}^e = E\left[X_{t+1} | \mathcal{F}_t\right]$$

– Define the rational expectations forecast error

$$\epsilon_{t+1} = X_{t+1} - E\left[X_{t+1} | \digamma_t\right]$$

– According to the REH then

$$E\left[\epsilon_{t+1} | \digamma_t\right] = 0$$

– Note: ϵ_{t+1} has zero expected value and is uncorrelated with any information available to the agents at time t

– In the absence of uncertainty REH corresponds to the perfect foresight assumption.

- Representative consumer maximises expected value of lifetime utility

$$U_t = E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} u(C_s) \right]$$

- Intertemporal budget constraint with transversality condition imposed

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} (C_s + I_s) = (1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} (Y_s - G_s)$$

- Optimisation problem

$$\max \ E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} u((1+r)B_s - Y_s - I_s - G_s + B_{s+1}) \right]$$

- First-order condition with respect to B_{s+1}

$$E_t \left[u'(C_s) \right] = \beta(1+r) E_t \left[u'(C_{s+1}\right])$$

for $s = t$

$$u'(C_t) = \beta(1+r) E_t \left[u'(C_{s+1}) \right]$$

- The linear-quadratic permanent income model

  - Assume that the period utility function is

$$u(C) = C - \frac{a_0}{2}C^2, \qquad a_0 > 0$$

  - Implies that marginal utility is linear

$$u'(C) = 1 - a_0 C$$

  - Stochastic Euler equation assuming $\beta = 1/(1+r)$

$$\begin{aligned} 1 - a_0 C_t &= 1 - a_0 E_t\left[C_{t+1}\right] \\ C_t &= E_t\left[C_{t+1}\right] \end{aligned}$$

– REH implies

$$C_{t+1} = E_t \left[C_{t+1} \right] + \varepsilon_{t+1}$$

and thus

$$C_{t+1} = C_t + \varepsilon_{t+1}$$

that is; consumption follows a random walk (Hall, 1978)

– Consumption function

$$E_t\left[\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}C_s\right]=E_t\left[(1+r)B_t+\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s-I_s-G_s)\right]$$

– From the Euler equation: $E_tC_s=C_t$

$$C_t=\frac{r}{1+r}\left[(1+r)B_t+\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}E_t\left(Y_s-I_s-G_s\right)\right]$$

– Certainty equivalence: people make decisions under uncertainty as if the future stochastic variables were sure to turn out equal to their conditional expectations (that is; uncertainty about future income has no impact on current consumption).

- The linear quadratic model and the current account

  - Consider an endowment economy without government spending

  - Output follows a first-order autoregressive (AR) process*

$$Y_t - \overline{Y} = \rho \left(Y_{t-1} - \overline{Y} \right) + \varepsilon_t$$

    where $0 \leq \rho \leq 1$ is a measure of the persistence of the process and $E[\varepsilon_t] = 0$ and $E[\varepsilon_t \varepsilon_s] = 0$ for $s \neq t$.

  - Impulse response function (or dynamic multiplier)

$$\frac{\partial Y_s}{\partial \varepsilon_t} = \rho^{s-t}$$

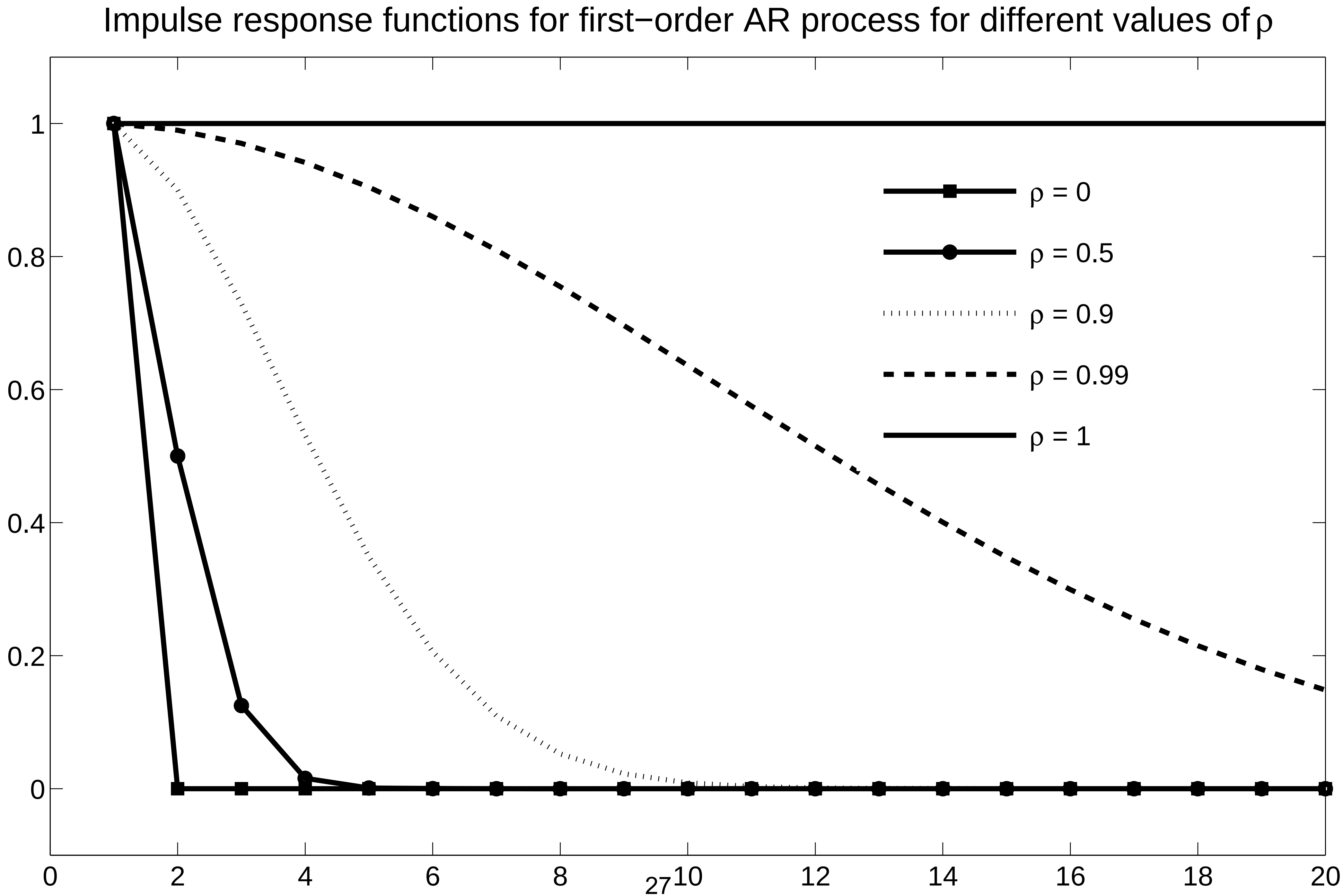
  - When $\rho = 0$, a shock has purely transitory effects on Y_t

  - When $\rho = 1$, a shock has permanent effect on Y_t (unit root process)

*See Hamilton (1994): Time Series Analysis, chapter 1 for details.

- By the *law of iterated conditional expectations* (the current expectation of a future expectation of a variable equals the current expectation of the variable, e.g., $E_t[E_{t+1}[\varepsilon_{t+2}]] = E_t[\varepsilon_{t+2}]$)

$$E_t\left[Y_s - \overline{Y}\right] = \rho^{s-t}\left(Y_t - \overline{Y}\right)$$


Impulse response functions for first−order AR process for different values of ρ
ρ = 0
ρ = 0.5
ρ = 0.9
ρ = 0.99
ρ = 1
1
0.8
0.6
0.4
0.2
0
0
2
4
6
8
10
12
14
16
18
20

– Insert into consumption function

$$
\begin{aligned}
C_t &= \frac{r}{1+r}\left[(1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} E_t Y_s\right] \\
&= \frac{r}{1+r}\left[(1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} \left(\rho^{s-t}\left(Y_t - \overline{Y}\right) + \overline{Y}\right)\right] \\
&= rB_t + \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\overline{Y} \\
&\quad + \frac{r}{1+r}\left[\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\rho^{s-t}\left(Y_t - \overline{Y}\right)\right] \\
&= rB_t + \overline{Y} + \frac{r}{1+r}\left[\sum_{s=t}^{\infty}\left(\frac{\rho}{1+r}\right)^{s-t}\right]\left(Y_t - \overline{Y}\right) \\
&= rB_t + \overline{Y} + \frac{r}{1+r}\left[\frac{1}{1-\frac{\rho}{1+r}}\right]\left(Y_t - \overline{Y}\right) \\
&= rB_t + \overline{Y} + \frac{r}{1+r-\rho}\left(Y_t - \overline{Y}\right)
\end{aligned}
$$

– For $\rho < 1$ a one unit increase in output increases consumption by less than one unit ('Keynesian' consumption function).

– Substitute in for $Y_t - \overline{Y} = \rho\left(Y_{t-1} - \overline{Y}\right) + \varepsilon_t$

$$
\begin{aligned}
C_t &= rB_t + \overline{Y} + \frac{r}{1+r-\rho}\left(\rho\left(Y_{t-1} - \overline{Y}\right) + \varepsilon_t\right) \\
&= rB_t + \overline{Y} + \frac{r\rho}{1+r-\rho}\left(Y_{t-1} - \overline{Y}\right) + \frac{r}{1+r-\rho}\varepsilon_t
\end{aligned}
$$

– Implications for the current account

$$
\begin{aligned}
CA_t &= rB_t + Y_t - C_t \\
&= rB_t + Y_t - \left(rB_t + \overline{Y} + \frac{r}{1+r-\rho}\left(Y_t - \overline{Y}\right)\right) \\
&= \frac{1-\rho}{1+r-\rho}\left(Y_t - \overline{Y}\right)
\end{aligned}
$$

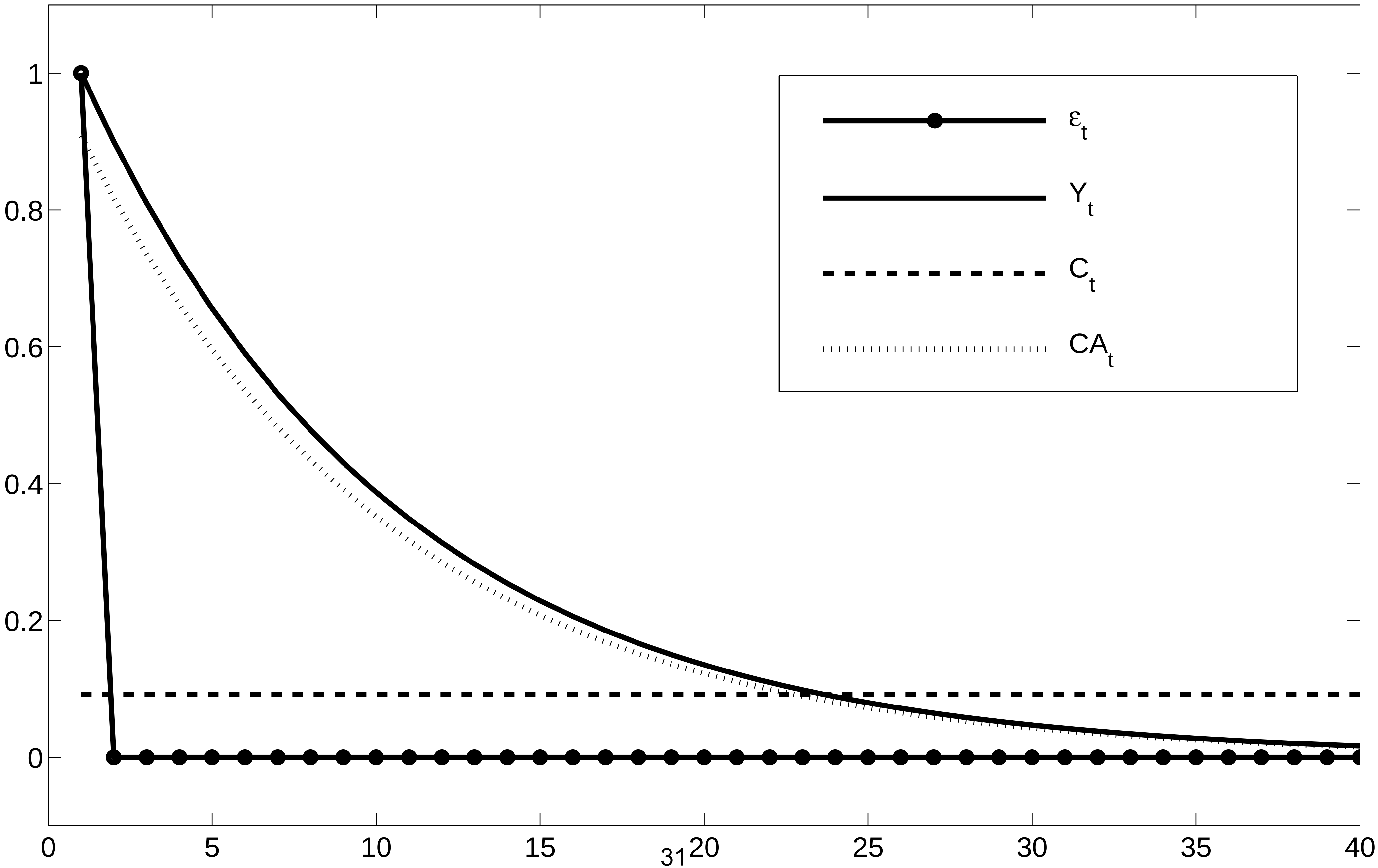
– Substitute in for $Y_t - \overline{Y} = \rho\left(Y_{t-1} - \overline{Y}\right) + \varepsilon_t$

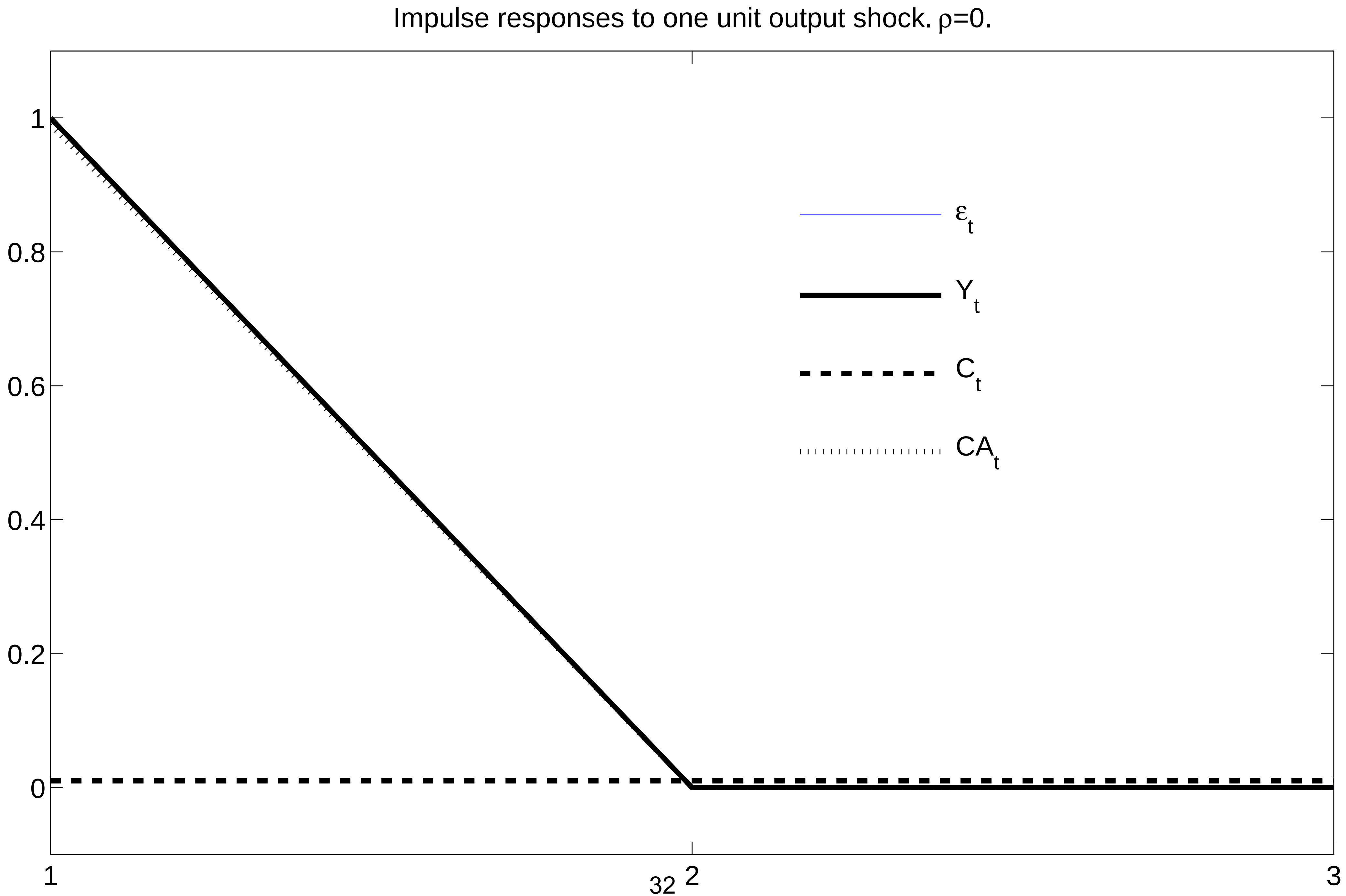
$$
\begin{aligned}
CA_t &= \frac{1-\rho}{1+r-\rho}\left(\rho\left(Y_{t-1} - \overline{Y}\right) + \varepsilon_t\right) \\
&= \rho\frac{1-\rho}{1+r-\rho}\left(Y_{t-1} - \overline{Y}\right) + \frac{1-\rho}{1+r-\rho}\varepsilon_t
\end{aligned}
$$

– An unexpected shock to output ($\varepsilon_t > 0$) causes a rise in the current account surplus as long as the shock is temporary ($\rho < 0$)

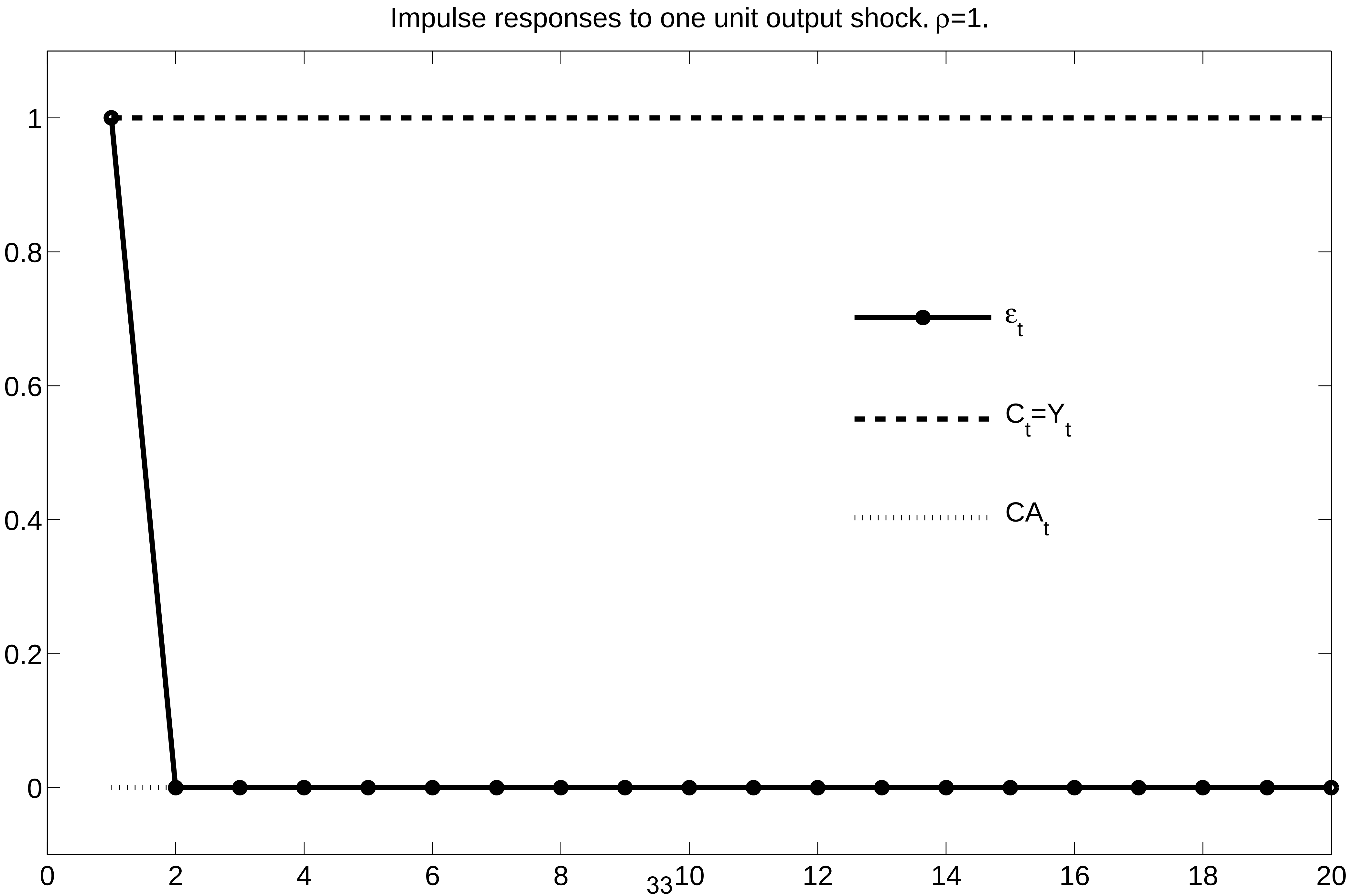
– Graphs show the impulse responses of Y_t, C_t and CA_t to a one unit shock to output in the linear quadratic model for different values of ρ when $B_1 = 0, Y_0 = \overline{Y} = 0, \beta = 0.99, r = (1-\beta)/\beta$.

Impulse responses to one unit shock to output. ρ=0.9.





Impulse responses to one unit output shock. ρ=0.
1
0.8
0.6
0.4
0.2
0
1
2
3
εt
Yt
Ct
CAt



Impulse responses to one unit output shock. ρ=1.
1
0.8
0.6
0.4
0.2
0
0
2
4
6
8
10
12
14
16
18
20
ε t
C t=Y t
CA t

```
clear all;
%Set parameters
P = 40; %Impulse response horizons
beta = 0.99; %Discount factor
r = (1-beta)/beta; %Exogenous real interest rate
YMEAN = 0; % Mean output level
rho = 0.9; % Persistence of output process
B(1) = 0; % Initial level of debt

%Shock process (one unit shock in period 1)
e = zeros(1,P);
e(1)=1;

%Output process (assuming Y(0)=YMEAN)
Y(1) = YMEAN+e(1);
for s = 2:P;
Y(s)=rho*(Y(s-1)-YMEAN)+YMEAN+e(s);
end


%Solution for consumption, net foreign assets and current account balance
for s = 1:P;
C(s) = r*B(s)+YMEAN+(r/(1+r-rho))*(Y(s)-YMEAN); %Consumption function
B(s+1)=(1+r)*B(s)+Y(s)-C(s); % Period s budget constraint
CA(s) = B(s+1)-B(s); % Definition of current account balance
end
```

- Does a positive output innovation always imply a current account surplus? NO!
  - Assume that output growth is positively serially correlated

$$Y_t - Y_{t-1} = \rho(Y_{t-1} - Y_{t-2}) + \varepsilon_t$$

  - Impulse responses

$$\frac{\partial Y_t}{\partial \varepsilon_t} = 1, \frac{\partial Y_{t+1}}{\partial \varepsilon_t} = 1 + \rho, \frac{\partial Y_{t+2}}{\partial \varepsilon_t} = 1 + \rho + \rho^2, \ldots, \lim_{s \to \infty} \frac{\partial Y_s}{\partial \varepsilon_t} = \frac{1}{1 - \rho}$$

  - Implication: Future output rises more than ε_t $\longrightarrow$ permanent output fluctuates more than current output $\longrightarrow$ unexpected increase in output leads to a larger increase in consumption and a current account deficit

- Deaton's paradox: In the data, output growth is positively serially correlated, yet consumption does not respond more than proportionally to output changes.

- Homework on Deaton's paradox: Exercise 4 in OR ch.2 (answers will be provided next week)